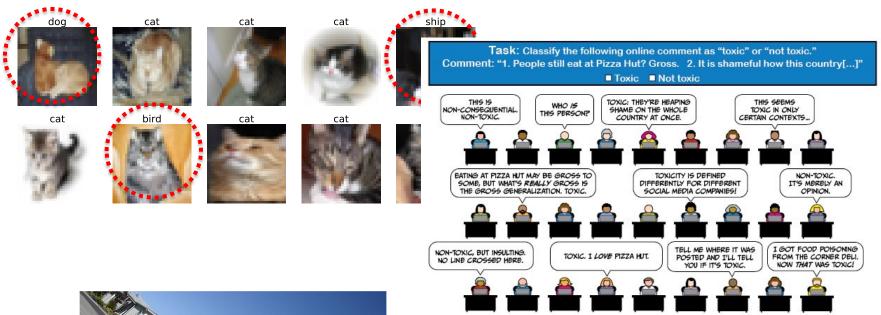
# Incentive Mechanisms for Data: The Peer Prediction Approach

### Yang Liu

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Summer School on Game Theory and Social Choice 2022

### Human annotations are prone to errors..





### **Preferred caption:**

A young boy wearing a orange

helmet riding a scooter.

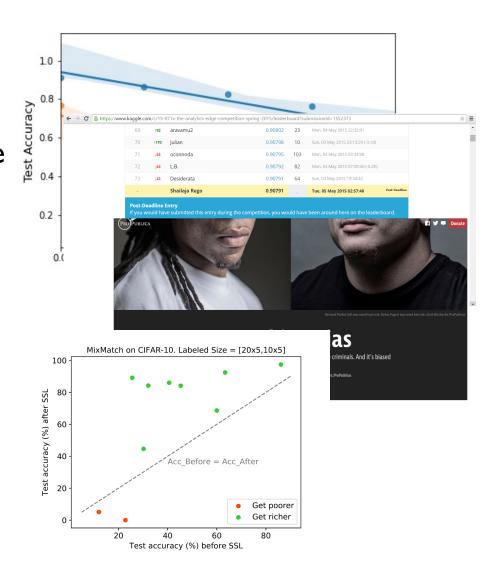
#### **Noisy caption:**

A person scating on the road.

# Harms of noisy annotations

- Harms training accuracy
- Invalidate model performance
- False sense of fairness
- Leads to biased treatments

..and more



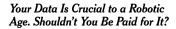
### **Incentives**



#### OCCAM'S PROFESSOR

"WHEN FACED WITH TWO POSSIBLE WAYS OF DOING SOMETHING, THE MORE COMPLICATED ONE IS THE ONE YOUR PROFESSOR WILL MOST LIKELY ASK YOU TO DO."

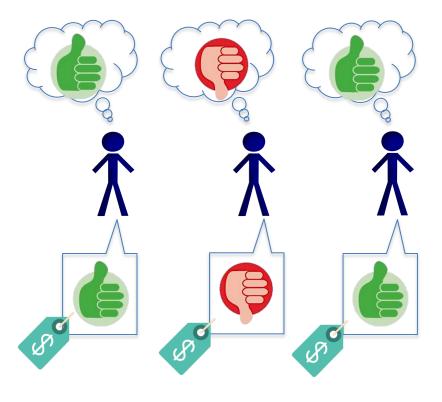




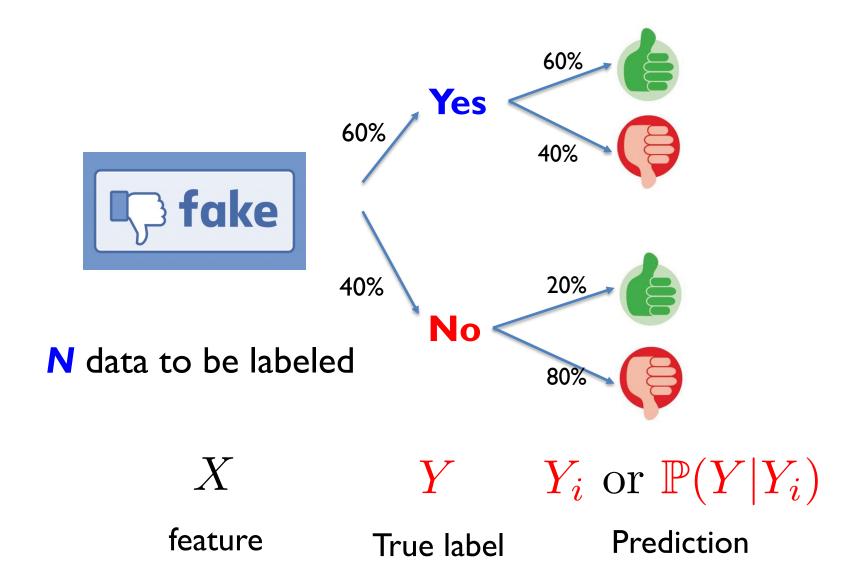


Eduardo Porter ECONOMIC SCENE MARCH 6, 2018 0000





### Running example



### Main Question

Can we incentivize high-quality prediction when the ground truth is unavailable?

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### Two meanings:

> Incentivize truthful reporting

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### Two meanings:

- > Incentivize truthful reporting
- > Accurate prediction get higher expected rewards

### Information Elicitation with Ground-truth

➤ How likely will it rain tomorrow?



➤ How likely will it rain tomorrow?



The principal pays:

SPSR:  $S(q_i, Y)$ Report of agent i

➤ How likely will it rain tomorrow?



The principal pays: Report of agent i SPSR:  $S(q_i, Y)$  Ground truth  $Y \in \{0,1\}$ 

➤ How likely will it rain tomorrow?



The principal pays: Report of agent i  $SPSR: S(q_i, Y)$ Ground truth  $Y \in \{0,1\}$ 

 $\triangleright$  Truthfulness:  $S(q_i, Y)$  is SPSR if and only if

$$\forall p_i, q_i \neq p_i, \mathbb{E}_{Y \sim p_i}[S(p_i, Y)] > \mathbb{E}_{Y \sim p_i}[S(q_i, Y)]$$

➤ How likely will it rain tomorrow?



Ground truth  $Y \in \{0,1\}$ 

The principal pays:

Report of agent iSPSR:  $S(q_i, Y)$ 

 $\triangleright$  Truthfulness:  $S(q_i, Y)$  is SPSR if and only if

$$\forall p_i, q_i \neq p_i, \mathbb{E}_{Y \sim p_i}[S(p_i, Y)] > \mathbb{E}_{Y \sim p_i}[S(q_i, Y)]$$

Belief of agent i

➤ How likely will it rain tomorrow?



The principal pays:

Report of agent i

SPSR:  $S(q_i, Y)$  Ground truth  $Y \in \{0,1\}$ 

 $\triangleright$  Truthfulness:  $S(q_i, Y)$  is SPSR if and only if

$$\forall p_i, q_i \neq p_i, \mathbb{E}_{Y \sim p_i}[S(p_i, Y)] > \mathbb{E}_{Y \sim p_i}[S(q_i, Y)]$$
 Belief of agent  $i$  From agent  $i$ 's perspective

> Example:  $S(q_i, Y) = 1 - (q_i - Y)^2$ 

- $\triangleright$  Example:  $S(q_i, Y) = 1 (q_i Y)^2$
- > Accurate predictions get higher rewards

- > Example:  $S(q_i, Y) = 1 (q_i Y)^2$
- > Accurate predictions get higher rewards
  - $> p^*$  true distribution of Y (fixed)

True belief: p = 0.6 for an event happening

Report a q in [0,1]

### **Expected scores**

$$0.6 \cdot (1 - (1 - q)^{2}) + 0.4 \cdot (1 - (0 - q)^{2})$$

$$= 1 - 0.6 \cdot (1 - 2q + q^{2}) - 0.4 \cdot q^{2}$$

$$= -q^{2} + 1.2q + 0.4$$

$$q^* = 0.6$$

### **Truth telling > Deviation**

## Challenges

Costly or impossible (e.g., significant delay) to verify reports against observable ground truth

- Label collection
- Peer review/grading
- Personal record
- Q: will people land on Mars by 2030?

**Peer Prediction**: A family of algorithms mechanisms to truthfully elicit private or high quality signals at equilibria

[Prelec 04, Miller et al. 05, Jurca & Faltings 09, Witkowski & Parkes 12, Radanovic & Faltings 13, Dasgupta & Ghosh 13, Shnayder et al. 16]



### Peer Prediction

### Key Idea: verify the reports against one another

- Reward = how well each report correlates with other reports
- Truthful equilibrium

Person A	Person B	Payment for A
		\$1.50
		\$0.10
		\$0.30
		\$1.20

A Scoring rule:  $S(q_A, q_B)$ 

## Design Goal

- $\triangleright$  A Scoring rule:  $S(q_A, q_B)$
- > Truthfulness at Bayesian Nash Equilibrium

$$\forall q_A \neq p_A, \mathbb{E}_{p_B|p_A}[S(p_A, p_B)] > \mathbb{E}_{p_B|p_A}[S(q_A, p_B)]$$

### Peer Prediction

> Reporting cateogrical signals

$$y_i, y_{-i} \in \{0,1\}$$

> Predicting peer agent's prediction

SPSR: 
$$S(q_i, y_{-i}), q_i = Pr(y_{-i}|y_i)$$

Eliciting Informative Feedback: The Peer-Prediction Method, Miller et al. 2005.

### Peer Prediction

SPSR: 
$$S(q_i, y_{-i}), q_i = Pr(y_{-i}|y_i)$$

- Inherits guarantees from SPSR at a Bayesian Nash Equilibrium
  - If everybody else is truthfully reporting, the best to do is also truthful reporting
- Caveat: Has to know the information structure to update

Eliciting Informative Feedback: The Peer-Prediction Method, Miller et al. 2005.

# Bayesian Truth Serum

"What is right is not always popular and what is popular is not always right."

--- Albert Einstein

Solution: Bayesian Truth Serum

- A surprisingly more popular answer is the correct answer
- Each participant also answers the question: how much they believe that others will agree with themselves (f)

BTS
$$(x = i, f) = \underbrace{\log \frac{\bar{x}_i}{\bar{f}_i}}_{\text{information score}} - \underbrace{\sum_{j} \bar{x}_j \log \frac{f_j}{\bar{x}_j}}_{\text{prediction penality}}$$

$$\bar{x}_i = \text{average of } (x_n = i)$$

$$\bar{f} = \text{geometric average of} (f_n)$$

# Multi-task Setting

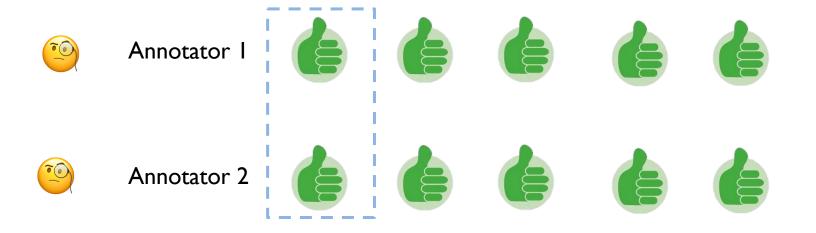
PP and BTS are single task mechanisms

> Each has limitations

Using correlation among tasks

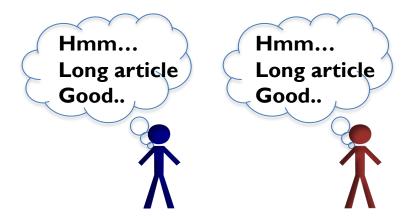
- $\triangleright$  N A set of agents (index i)
- $\rightarrow M A$  set of tasks (index k)
- $\succ Y_k \in \{0,1\}$  the ground truth of task k
- $ightharpoonup Z_{i,k}$  the report of agent i on task k

# What went wrong with agreement



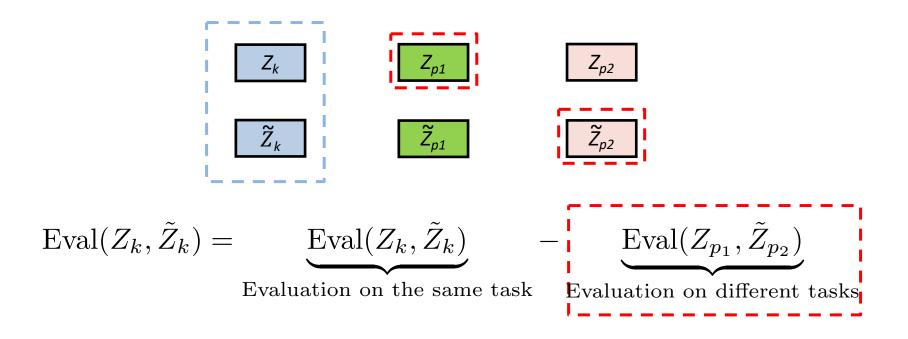
Using agreement metric = I e

# Cheap signal

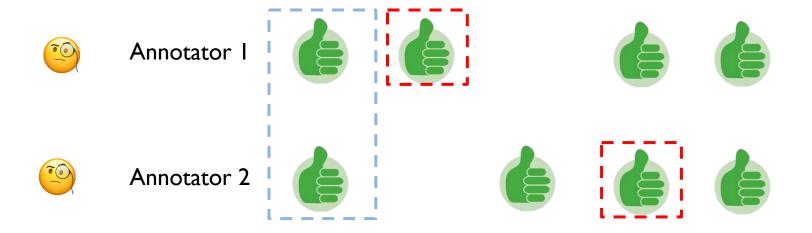


- Cheap signal for collusion
- Common mistakes
- Better equilibrium for agents to follow

## Correlated Agreement



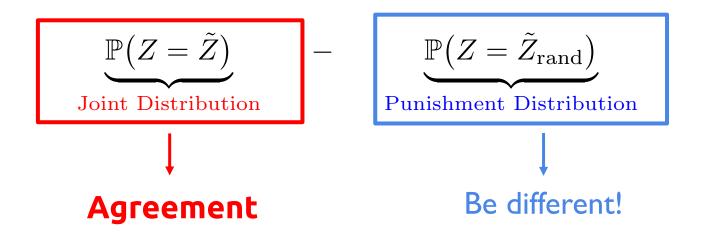
### Correlated Agreement



Correlated agreement = I - I = 0

### Why CA works?

Expectation of CA calibrates:



**Theorem:** CA induces truthful report at a Bayesian Nash Equilibrium.

### ML aided Peer Prediction

$$S(y_i^{report}, ML)$$

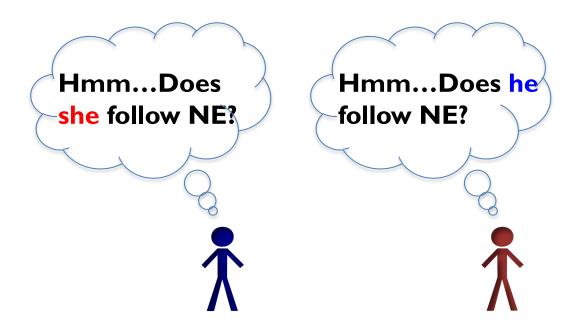
$$f(\mathbf{x}) \to \mathbf{y}$$

$$\mathbb{E}[S(\tilde{Y}_i, \mathrm{ML}(\mathcal{I}_{-i}))] > \mathbb{E}[S(Z_i \neq \tilde{Y}_i, \mathrm{ML}(\mathcal{I}_{-i}))]$$

- No ground-truth verification
   Predict via ML
   No receive No requirement of others' report

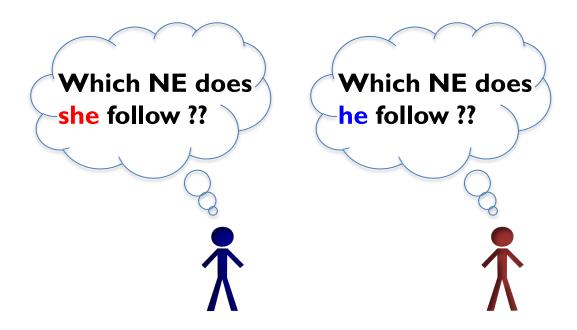
### Caveats and What We Want

### **Equilibria** v.s. **Dominant Truthfulness**

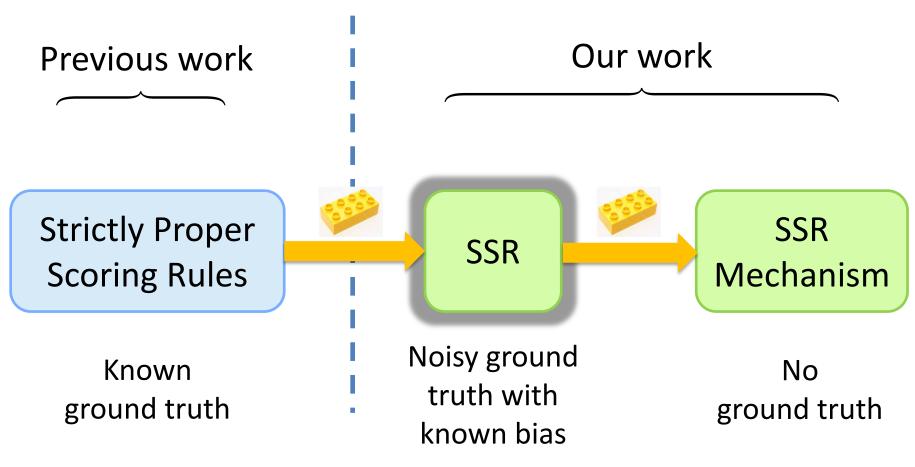


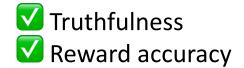
### Caveats and What We Want

### Equilibria v.s. Dominant Truthfulness



# Surrogate Scoring Rules (SSR)





Surrogate Scoring Rules. Liu et al. EC 2020.

# Surrogate Scoring Rules (SSR)

$$R(q_i, Z; e_z^+, e_z^-)$$

A noisy ground truth with known error rates:

- $e_z^+$ : =  $\Pr(Z=0|Y=1)$  (False negative rate)
- $e_z^-$ : = Pr(Z = 1|Y = 0) (False positive rate)

# Surrogate Scoring Rules (SSR)

$$R(q_i, Z; e_z^+, e_z^-)$$

A noisy ground truth with known error rates:

- $e_z^+$ : =  $\Pr(Z = 0 | Y = 1)$  (False negative rate)
- $e_z^-$ : = Pr(Z = 1|Y = 0) (False positive rate)
- Unbiasedness property (how SSR are defined):

$$\mathbb{E}_{\mathbf{Z}}[R(q_i, \mathbf{Z}; e_z^+, e_z^-)] = \mathbb{E}_{\mathbf{Y}}[S(q_i, \mathbf{Y})], \forall q_i$$

# An implementation of SSR

$$R(q_i, Z = 1) = \frac{(1 - e_Z^-) \cdot S(q_i, 1) - e_Z^+ \cdot S(q_i, 0)}{1 - e_Z^- - e_Z^+}$$

$$R(q_i, Z = 0) = \frac{-e_Z^+ \cdot S(q_i, 1) + (1 - e_Z^-) \cdot S(q_i, 0)}{1 - e_Z^- - e_Z^+}$$

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Degenerate to SPSR when  $e_z^- = e_z^+ = 0$ 

# An implementation of SSR

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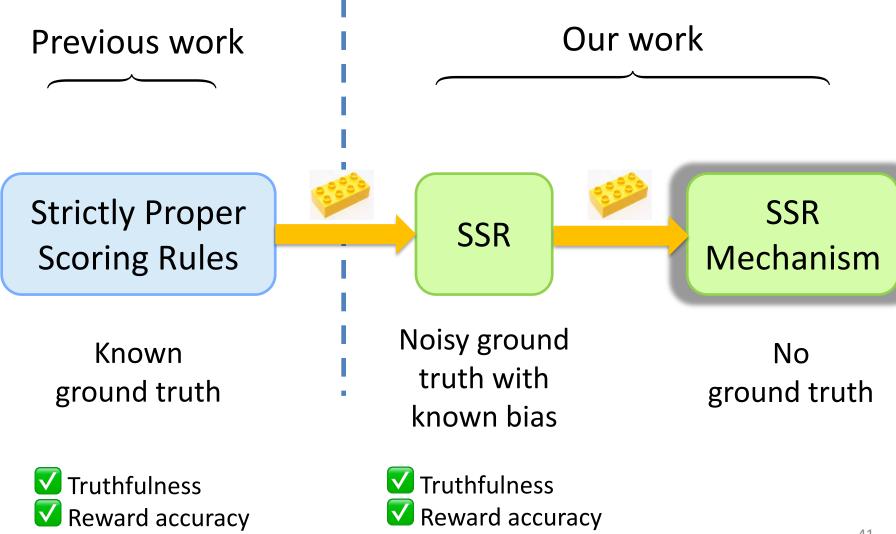
$$R(q_i, Z = 0) = \frac{-e_Z^+ \cdot S(q_i, 1) + (1 - e_Z^-) \cdot S(q_i, 0)}{1 - e_Z^- - e_Z^+}$$

> Weights are designed for unbiasedness property:

Lemma 1. For this implementation,

$$\mathbb{E}_{Z|Y}[R(q_i, Z; e_z^+, e_z^-)] = S(q_i, Y), \forall q_i, Y$$

# Roadmap



> N - A set of agents (index i)

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- > M A set of tasks (index k)
- $\succ Y_k \in \{0,1\}$  the ground truth of task k
- $ightharpoonup p_{i,k}$ ,  $q_{i,k}$  the belief/report of agent i on task k
- Each task is assigned to at least 3 agents

When score a prediction  $q_{i,k}$ :

SSR: 
$$R(q_{i,k}, Z; e_z^+, e_z^-)$$

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SSR: 
$$R(q_{i,k}, Z; e_z^+, e_z^-)$$

- ➤ Construct *Z*
- $\triangleright$  Estimate  $e_z^+, e_z^-$
- > Apply SSR

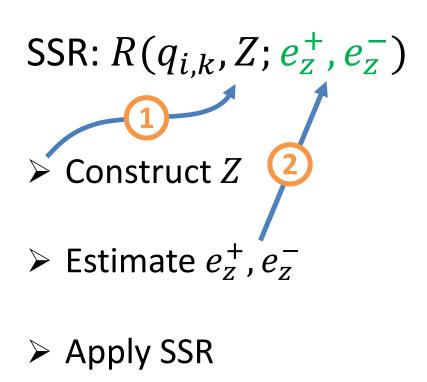
When score a prediction  $q_{i,k}$ :

SSR:  $R(q_{i,k}, \mathbf{Z}; e_z^+, e_z^-)$ 

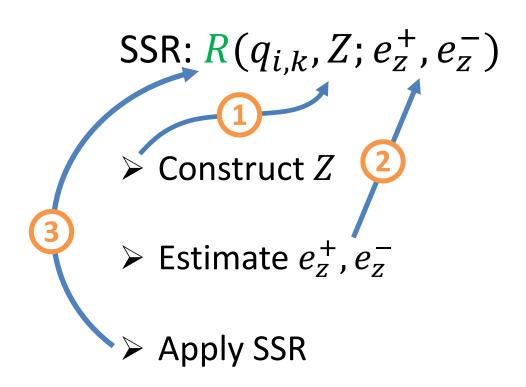


- $\triangleright$  Construct Z
- $\triangleright$  Estimate  $e_z^+, e_z^-$
- > Apply SSR

When score a prediction  $q_{i,k}$ :



When score a prediction  $q_{i,k}$ :

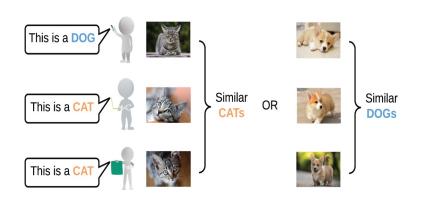


### Construct Z

For a task k, uniformly randomly pick an agent  $j \neq i$ , draw Z=1 with probability  $q_{j,k}$  the report of agent j on task k

#### HOC

#### Using high-order consensus to infer the noise transition matrix



Surrogate Scoring Rules, ACM EC 2020. **Liu**, Wang and Chen.

Clusterability as an Alternative to Anchor Points When Learning with Noisy Labels, ICML 2021. Zhu, Song, Liu. Best paper award at IJCAI workshop on weakly supervised representation learning.



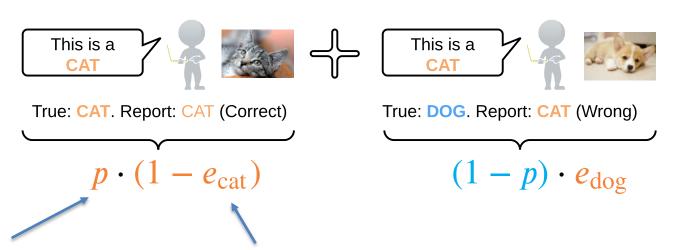
Solver and Implementation: https://github.com/UCSC-REAL/HOC

# Calculate the probability

- Binary classification: Cat or Dog
- 1st-order (2 patterns)

Pattern "CAT"

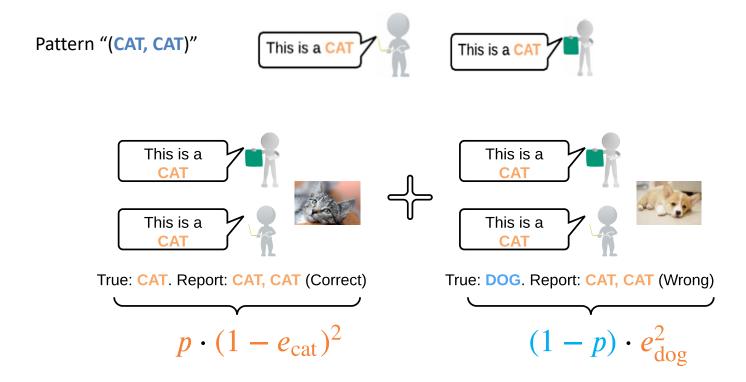
This is a CAT



Population of true Cat Noise rate of class Cat

# Calculate the probability (Binary example)

2nd-order (4 patterns)



# Calculate the probability (Binary example)

3rd-order (8 patterns)

Pattern "(DOG, CAT, CAT)"

This is a DOG

This is a CAT

This is a DOG

This is a CAT

True: CAT. Report: DOG, CAT, CAT

True: DOG, Report: DOG, CAT, CAT

True: DOG, Report: DOG, CAT, CAT

 $p \cdot e_{\text{cat}} \cdot (1 - e_{\text{cat}})^2$   $(1 - p) \cdot (1 - e_{\text{dog}}) \cdot e_{\text{dog}}^2$ 

### High-Order Consensuses (HOC)

#### **Consensus Equations**

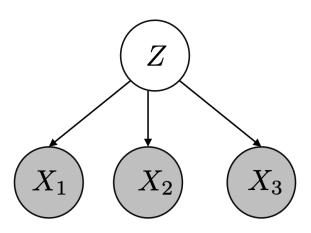
- ullet 1st-order (K equations):  $oldsymbol{c}^{[1]} := oldsymbol{T}^{ op} oldsymbol{p}$
- 2nd-order ( $K^2$  equations):  $\boldsymbol{c}_r^{[2]} := (\boldsymbol{T} \circ \boldsymbol{T}_r)^{\top} \boldsymbol{p}, \ r \in [K]$
- 3rd-order ( $K^3$  equations):  $\boldsymbol{c}_{r,s}^{[3]} := (\boldsymbol{T} \circ \boldsymbol{T}_r \circ \boldsymbol{T}_s)^{\top} \boldsymbol{p}, \ r, s \in [K]$

T :=Noise transition matrix

### Three noisy labels are necessary and sufficient

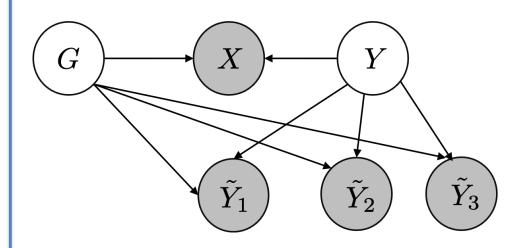
#### Kruscal's Identifiability results:

the identifiability of unobserved model  $Z \rightarrow X$  relies on the informativeness of three observations X.



Kruskal, J. B. Linear algebra and its applications, 18(2):95–138, 1977.

Our theorems: (I) Three labels are necessary and sufficient at instance level (2) Informative features help too.



Identifiability of Label Noise Transition Matrix, Liu, 2022.

# Apply SSR

SSR property: 
$$\mathbb{E}_{Z|Y_k}[R(p_{i,k},Z;\widehat{e_z^+},\widehat{e_z^-})] = S(p_{i,k},Y_k)$$

SSR mechanisms inherit two properties of SPSR:

- > Incentivizing truthful reporting
- > Accurate predictions get higher rewards

Theorem 1. Under A1 $^{\sim}$ A4, in SSR mechanisms, truthful reporting is the uniform dominant strategy when M, N  $\rightarrow \infty$ 

# **Apply SSR**

SSR property: 
$$\mathbb{E}_{Z|Y_k}[R(p_{i,k},Z;\widehat{e_z^+},\widehat{e_z^-})] = S(p_{i,k},Y_k)$$

SSR mechanisms inherit two properties of SPSR:

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Theorem 1. Under A1 $^{\sim}$ A4, in SSR mechanisms, truthful reporting is the uniform dominant strategy when M, N  $\rightarrow \infty$ 

$$\epsilon \sim O\left(\frac{1}{N} + \frac{1}{\sqrt{M}}\right)$$

### Other challenges & extensions

# Learning to design optimal mechanism

### Workers are effort sensitive $e_i \in \{H, L\}$

> Exerting effort leads to better data

$$p_{i,e_i} = \Pr(s' = s | s, e_i) \quad 1 \ge p_H > p_L \ge 0.5$$

> Exerting effort incurs unknown cost

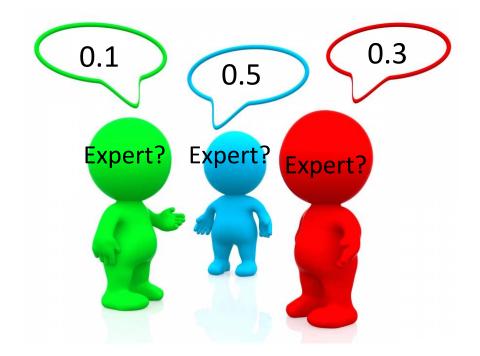
$$c \in F(c), c \in [0, c_{\text{max}}]$$

Goal: learnig to design best payment mechanism

Sequential Peer Prediction: Learning to Elicit Effort using Posted Prices. Liu and Chen. AAAI 2017.

# Aggregation using peer prediction

Use peer prediction mechanisms to identify experts



Forecast Aggregation via Peer Prediction. Wang et al. HCOMP 2021.

# Online Aggregation Using Peer Prediction

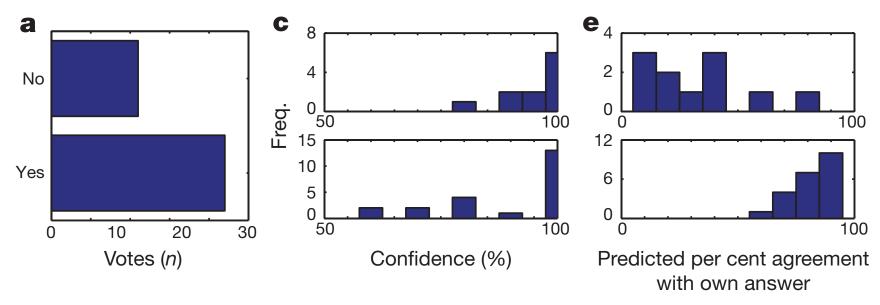
- Maintain a set of weights for each agent  $w_i(t)$
- Aggregate:  $\sum_{i=1}^{N} \frac{w_i(t)}{\sum_i w_j(t)} p_i(t)$
- At each round t, observe outcome of the event; each agent i incurs a score  $(S_t(p_i(t), y_t))$
- Update weight of i using  $w_i(t+1) := w_i(t) \cdot (1 + \eta \cdot S_t(p_i(t), y_t))$

How to compute the weights w.o.  $y_t$ ??

Online Learning Using Only Peer Prediction. Liu and Helmbold. AISTATS 2020.

### Aggregation Using BTS

Philadelphia is the capital of Pennsylvania, yes or no?



Answer "No" received: [Vote: ~30%, Peer Prediction: ~20%] 30% < 50%, but

30% > 20% <= surprisingly more popular

We are developed a machine learning version of this method.

### **Takeaways**

- A lot of practical challenges
  - budget constraints
  - human-level information structure
- A lot of practical concerns
  - Interpretability of the mechanisms
- Human-in-the-loop => Machine-in-the-loop

# Questions?

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# Aggregation Using BTS

An answer is the correct answer only if (surprisingly popular)

Percentage of the answer > Percentage of peer predicted answer

Is this a fake news?

What's your answer? How many others would agree with you?

Expert I: (NO, 20%) (I know I have the minority answer)

Expert 2: (YES,70%) (Easy, most ppl know)

Expert 3: (YES, 80%) (Easy, most ppl know)

NO: 0.3333 > (20% + 30% + 20%)/3 = 0.2333 (TRUTH)

YES: 0.6666 < (0.8+0.7+0.8)/3 = 0.7777